Probability

If an experiment can result in any one of N different equally likely events/outcomes, and if n of these outcomes together constitute event A, then the probability of event A is

$$P(A) = \frac{n}{N}$$

Some Rules of Probability

1. $P(A) \geq 0$
2. $P(S) = 1$ Probability of sample space
3. $P(\phi) = 0$
4. $0 \leq P(A) \leq 1$ For any event A
5. $P(\overline{A}) = 1 - P(A)$
6. If $A_1, A_2, A_3, \ldots$ is a finite or infinite sequenece of mutually exclusive events of S, then

   $P(A_1 \cup A_2 \cup A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \cdots$

   $P(A \cap B) = 0 \Rightarrow A$ and B are mutually exclusive or disjoint
7. If A and B are events in a sample space S and $A \subset B$, then

   $P(A) \leq P(B)$
8. **General Addition Rule** If A and B are any two events in a sample space S, then the probability of two events A or B or both is

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   For three events

   $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$

Odds

The odds that an event will occur are given by the ratio of the probability that an event will occur to the probability that it will not occur, provided that neither probability is zero. If a to b $\left(\frac{a}{b}\right)$ are the odds that an event will occur, then its probability is

$$p = \frac{a}{a+b}$$

and of not occurrence is

$$q = \frac{b}{a+b}$$

Conditional Probability

If A and B are any two events in a sample space S and $P(A) \neq 0$, then the probability that event B occurred when it is known that event A occurred

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

This implies the multiplication rule i.e the probability of occurrence of both events A and B

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$$

Moreover

$$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$$

Independent Events

Two events A and B are independent if the probability of occurrence of the event A is not effected by the occurrence or non occurrence of event B, and vice versa. In terms conditional probabilities it implies

$$P(A/B) = P(A) \quad \text{and} \quad P(B/A) = P(B)$$

For independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

Theorem: If A and B are independent, then A and B' are also independent, and so are A' and B, and A' and B'.

Generalization: Events $A_1, A_2, \ldots A_k$ are independent if and only if the probability of intersection of any $2, 3, \ldots$ or k of these events equals the probability of their respective probabilities.

For three events A, B and C, independence requires that

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ P(A \cap C) &= P(A) \cdot P(C) \\ P(B \cap C) &= P(B) \cdot P(C) \\ P(A \cap B \cap C) &= P(A) \cdot P(B) \cdot P(C) \end{aligned}$$

The general multiplication rule for independent events is

$$P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) = \prod_{i=1}^{n} P(A_i)$$

Rule of Total Probability

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i = 1, 2, 3, \ldots, k$, then for any event A in S

$$P(A) = \sum_{i=1}^{k} P(B_i) \cdot P(A/B_i)$$

Bays' Theorem

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i = 1, 2, 3, \ldots, k$, then for nay event A in S, such that $P(A) \neq 0$

$$P(B_r/A) = \frac{P(B_r) \cdot P(A/B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A/B_i)} \qquad k = 1, 2, \ldots, k$$

The Bays' theorem relates conditional and unconditional probabilities for events A and B through the multiplication rule. Using the symmetric form of multiplication rule

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$$

$$\Rightarrow P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$