Counting Rules

Multiplication Rule

If you are drawing one element from each of (different) k sets of elements, with the size of the set $n_1, n_2, \ldots, n_k$, the number of different results is

$$n_1 n_2 n_3 \cdots n_k$$

Permutations Rule

If you are drawing r elements from a set of n elements and arranging the n elements in a distinct order, the number of different results is

$$P_r^n = \frac{n!}{(n-r)!}$$

Partitions Rule

If you are partitioning the elements of a set of N elements into k groups consisting of $n_1, n_2, \ldots n_k$ elements $(n_1 + n_2 + \cdots n_k = N)$, the number of different results is

OR

Number of permutations of N objects of which n_1 are of first kind, n_2 of a second kind, $\cdots$, n_k of a kth kind and $(n_1 + n_2 + \cdots n_k = N)$, is

$$\frac{N!}{n_1! n_2! n_3! \cdots n_k!} = \binom{N}{n_1, n_2, \ldots, n_k}$$

Combinations Rule

If you are drawing r elements from a set of n elements without regard to the order of the elements, the number of different results is

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P_n^r}{r!}$$

Note: Combinations rule is a special case of the partitions rule with $k = 2$.