# Binomial and Multinomial Coefficients

## Stirling's Formula

For a large value of $n$, $n!$ can be approximated as

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \qquad \text{for large } n$$

## Binomial Coefficients

The binomial coefficients in the expansion of $(x+y)^n$ are

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r \qquad n \text{ is any positive integer}$$

If $n$ is a positive integer or zero, and $-1 < y < 1$, then

$$(1+y)^n = 1 + \binom{n}{1} y + \binom{n}{2} y^2 + \binom{n}{3} y^3 + \cdots + \binom{n}{r} y^r + \cdots$$

where

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} \qquad (n, r = 0, 1, 2, 3, \dots)$$

## Multinomial Coefficients

The multinomial coefficients of the terms $x_1^{r_1}, x_2^{r_2}, x_3^{r_3}, \dots, x_k^{r_k}$ in the expression/expansion of $(x_1 + x_2 + \cdots x_n)^n$ is

$$\binom{n}{r_1, r_2, \dots r_k} = \frac{n!}{r_1! r_2! r_3! \cdots r_k!}$$

## Permutation Relations

$$\binom{n}{0} = \binom{n}{n} = 1$$

$$\binom{n}{1} = \binom{n}{n-1} = n$$

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

$$\sum_{r=0}^{k} \binom{m}{r} \binom{n}{k-r} = \binom{m+n}{k}$$