

Sums of Powers of the First n Integers

$$\begin{aligned}\sum_{k=1}^n k &= 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \\ \sum_{k=1}^n k^2 &= 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \\ \sum_{k=1}^n k^3 &= 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} \\ \sum_{k=1}^n k^4 &= \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1) \\ \sum_{k=1}^n k^5 &= \frac{n^2}{12}(n+1)^2(2n^2+2n-1) \\ \sum_{k=1}^n k^6 &= \frac{n}{42}(n+1)(2n+1)(3n^4+6n^3-3n+1) \\ \sum_{k=1}^n k^7 &= \frac{n^2}{24}(n+1)^2(3n^4+6n^3-n^2-4n+2) \\ \sum_{k=1}^n k^8 &= \frac{n}{90}(n+1)(2n+1)(5n^6+15n^5+5n^4-15n^3-n^2+9n-3) \\ \sum_{k=1}^n k^9 &= \frac{n^2}{20}(n+1)^2(2n^6+6n^5+n^4-8n^3+n^2+6n-3) \\ \sum_{k=1}^n k^{10} &= \frac{n}{66}(n+1)(2n+1)(3n^8+12n^7+8n^6-18n^5-10n^4+24n^3+2n^2-15n+5)\end{aligned}$$