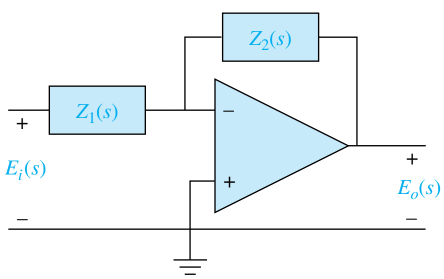
# Laplace Transform Table

| Laplace Transform $F(s)$ | Time Function $f(t)$ |
|---|---|
| $1$ | Unit-impulse function $\delta(t)$ |
| $\frac{1}{s}$ | Unit-step function $u_s(t)$ |
| $\frac{1}{s^2}$ | Unit-ramp function $t$ |
| $\frac{n!}{s^{n+1}}$ | $t^n (n = \text{positive integer})$ |
| $\frac{1}{s + \alpha}$ | $e^{-\alpha t}$ |
| $\frac{1}{(s + \alpha)^2}$ | $te^{-\alpha t}$ |
| $\frac{n!}{(s + \alpha)^{n+1}}$ | $t^n e^{-\alpha t} (n = \text{positive integer})$ |
| $\frac{1}{(s + \alpha)(s + \beta)}$ | $\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})(\alpha \neq \beta)$ |
| $\frac{s}{(s + \alpha)(s + \beta)}$ | $\frac{1}{\beta - \alpha}(\beta e^{-\beta t} - \alpha e^{-\alpha t})(\alpha \neq \beta)$ |
| $\frac{1}{s(s + \alpha)}$ | $\frac{1}{\alpha}(1 - e^{-\alpha t})$ |
| $\frac{1}{s(s + \alpha)^2}$ | $\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$ |
| $\frac{1}{s^2(s + \alpha)}$ | $\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$ |
| $\frac{1}{s^2(s + \alpha)^2}$ | $\frac{1}{\alpha^2}\left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$ |
| $\frac{s}{(s + \alpha)^2}$ | $(1 - \alpha t)e^{-\alpha t}$ |

| Laplace Transform $F(s)$ | Time Function $f(t)$ |
|---|---|
| $\dfrac{\omega_n}{s^2 + \omega_n^2}$ | $\sin \omega_n t$ |
| $\dfrac{s}{s^2 + \omega_n^2}$ | $\cos \omega_n t$ |
| $\dfrac{\omega_n^2}{s(s^2 + \omega_n^2)}$ | $1 - \cos \omega_n t$ |
| $\dfrac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$ | $\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$<br>where $\theta = \tan^{-1}(\omega_n/\alpha)$ |
| $\dfrac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$ | $\dfrac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \dfrac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$<br>where $\theta = \tan^{-1}(\omega_n/\alpha)$ |
| $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\dfrac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2}\, t \qquad (\zeta < 1)$ |
| $\dfrac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ | $1 - \dfrac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$<br>where $\theta = \cos^{-1} \zeta \qquad (\zeta < 1)$ |
| $\dfrac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\dfrac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t - \theta)$<br>where $\theta = \cos^{-1} \zeta \qquad (\zeta < 1)$ |
| $\dfrac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\omega_n \sqrt{\dfrac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}}\, e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$<br>where $\theta = \tan^{-1} \dfrac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \qquad (\zeta < 1)$ |
| $\dfrac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ | $t - \dfrac{2\zeta}{\omega_n} + \dfrac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$<br>where $\theta = \cos^{-1}(2\zeta^2 - 1) \qquad (\zeta < 1)$ |

## Inverting Op-Amp Transfer Functions

| | Input Element | Feedback Element | Transfer Function | Comments |
|---|---|---|---|---|
| (a) | $R_1$ <br> $Z_1 = R_1$ | $R_2$ <br> $Z_2 = R_2$ | $-\frac{R_2}{R_1}$ | Inverting gain. e.g., if $R_1 = R_2$, $e_o = -e_1$. |
| (b) | $R_1$ <br> $Z_1 = R_1$ | $C_2$ <br> $Y_2 = sC_2$ | $\left(\frac{-1}{R_1C_2}\right)\frac{1}{s}$ | Pole at the origin. i.e., an integrator. |
| (c) | $C_1$ <br> $Y_1 = sC_1$ | $R_2$ <br> $Z_2 = R_2$ | $(-R_2C_1)s$ | Zero at the origin. i.e., a differentiator. |
| (d) | $R_1$ <br> $Z_1 = R_1$ | $R_2$, $C_2$ <br> $Y_2 = \frac{1}{R_2} + sC_2$ | $\frac{-\frac{1}{R_1C_2}}{s + \frac{1}{R_2C_2}}$ | Pole at $\frac{-1}{R_2C_2}$ with a dc gain of $-R_2/R_1$. |
| (e) | $R_1$ <br> $Z_1 = R_1$ | $R_2$, $C_2$ <br> $Z_2 = R_2 + \frac{1}{sC_2}$ | $\frac{-R_2}{R_1}\left(\frac{s + 1/R_2C_2}{s}\right)$ | Pole at the origin and a zero at $-1/R_2C_2$, i.e., a PI Controller. |
| (f) | $R_1$, $C_1$ <br> $Y_1 = \frac{1}{R_1} + sC_1$ | $R_2$ <br> $Z_2 = R_2$ | $-R_2C_1\left(s + \frac{1}{R_1C_1}\right)$ | Zero at $s = \frac{-1}{R_1C_1}$, i.e., a PD controller. |
| (g) | $R_1$, $C_1$ <br> $Y_1 = \frac{1}{R_1} + sC_1$ | $R_2$, $C_2$ <br> $Y_2 = \frac{1}{R_2} + sC_2$ | $\frac{\frac{-C_1}{C_2}\left(s + \frac{1}{R_1C_1}\right)}{s + \frac{1}{R_2C_2}}$ | Pole at $s = \frac{-1}{R_2C_2}$ and a zero at $s = \frac{-1}{R_1C_1}$, i.e., a lead or lag controller. |



### Properties of the Root Loci of $F(s) = 1 + KG_1(s)H_1(s) = 0$

1. $K = 0$ points — The $K = 0$ points are at the poles of $G(s)H(s)$, including those at $s = \infty$.

2. $K = \pm\infty$ points — The $K = \pm\infty$ points are at the zeros of $G(s)H(s)$, including those at $s = \infty$.

3. Number of separate root loci — The total number of root loci is equal to the order of the equation $F(s) = 0$.

4. Symmetry of root loci — The root loci are symmetrical about the axes of symmetry of the pole-zero configuration of $G(s)H(s)$.

5. Asymptotes of root loci as $s \to \infty$ — For large values of $s$, the root loci for $K > 0$ are asymptotic to asymptotes with angles given by

$$\theta_i = \frac{2i + 1}{|n - m|} \times 180°$$

For $K < 0$, the root loci are asymptotic to

$$\theta_i = \frac{2i}{|n - m|} \times 180°$$

where $i = 0, 1, 2, \ldots, |n - m| - 1$,
$n$ = number of finite poles of $G(s)H(s)$, and
$m$ = number of finite zeros of $G(s)H(s)$.

6. Intersection of the asymptotes — (a) The intersection of the asymptotes lies only on the real axis in the $s$-plane.
(b) The point of intersection of the asymptotes is given by

$$\sigma_1 = \frac{\Sigma \text{ real parts of poles of } G(s)H(s) - \Sigma \text{ real parts of zeros of } G(s)H(s)}{n - m}$$

7. Root loci on the real axis — Root loci for $K > 0$ are found in a section of the real axis only if the total number of real poles and zeros of $G(s)H(s)$ to the **right** of the section is **odd**. If the total number of real poles and zeros to the right of a given section is **even**, root loci for $K < 0$ are found.

8. Angles of departure — The angle of departure or arrival of the root loci from a pole or a zero of $G(s)H(s)$ can be determined by assuming a point $s_1$ that is very close to the pole, or zero, and applying the equation

$$\angle G(s_1)H(s_1) = \sum_{k=1}^{m} \angle(s_1 + z_k) - \sum_{j=1}^{n} \angle(s_1 + p_j)$$
$$= 2(i + 1)180° \qquad K > 0$$
$$= 2i \times 180° \qquad K < 0$$

where $i = 0, \pm 1, \pm 2, \ldots$.

9. Intersection of the root loci — The crossing points of the root loci on the imaginary axis and with the imaginary axis the corresponding values of $K$ may be found by use of the Routh-Hurwitz criterion.

10. Breakaway points — The breakaway points on the root loci are determined by finding the roots of $dK/ds = 0$, or $dG(s)H(s)/ds = 0$. These are necessary conditions only.

11. Calculation of the values of $K$ — The absolute value of $K$ at any point $s_1$ on the root loci is on the root loci determined from the equation

$$|K| = \frac{1}{|G_1(s_1)H_1(s_1)|}$$

# $z$-Transform Table

| Laplace Transform | Time Function | $z$-Transform |
|---|---|---|
| $1$ | Unit impulse $\delta(t)$ | $1$ |
| $\frac{1}{s}$ | Unit step $u_s(t)$ | $\frac{z}{z-1}$ |
| $\frac{1}{1-e^{-Ts}}$ | $\delta_T(t) = \sum_{n=0}^{\infty} \delta(t-nT)$ | $\frac{z}{z-1}$ |
| $\frac{1}{s^2}$ | $t$ | $\frac{Tz}{(z-1)^2}$ |
| $\frac{1}{s^3}$ | $\frac{t^2}{2}$ | $\frac{T^2 z(z+1)}{2(z-1)^3}$ |
| $\frac{1}{s^{n+1}}$ | $\frac{t^n}{n!}$ | $\lim_{\alpha \to 0} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial \alpha^n} \left[ \frac{z}{z-e^{-\alpha T}} \right]$ |
| $\frac{1}{s+\alpha}$ | $e^{-\alpha t}$ | $\frac{z}{z-e^{-\alpha T}}$ |
| $\frac{1}{(s+\alpha)^2}$ | $te^{-\alpha t}$ | $\frac{Tze^{-\alpha T}}{(z-e^{-\alpha T})^2}$ |
| $\frac{\alpha}{s(s+\alpha)}$ | $1-e^{-\alpha t}$ | $\frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$ |
| $\frac{\omega}{s^2+\omega^2}$ | $\sin \omega t$ | $\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$ |
| $\frac{\omega}{(s+\alpha)^2+\omega^2}$ | $e^{-\alpha t} \sin \omega t$ | $\frac{ze^{-\alpha T} \sin \omega T}{z^2 - 2ze^{-\alpha T} \cos \omega T + e^{-2\alpha T}}$ |
| $\frac{s}{s^2+\omega^2}$ | $\cos \omega t$ | $\frac{z(z-\cos \omega T)}{z^2 - 2z \cos \omega T + 1}$ |
| $\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$ | $e^{-\alpha t} \cos \omega t$ | $\frac{z^2 - ze^{-\alpha T} \cos \omega T}{z^2 - 2ze^{-\alpha T} \cos \omega T + e^{-2\alpha T}}$ |