SERIES

Powers of Natural Numbers

$$\sum_{k=1}^{n} k = \frac{1}{2}\, n(n+1)\ ; \quad \sum_{k=1}^{n} k^2 = \frac{1}{6}\, n(n+1)(2n+1); \sum_{k=1}^{n} k^3 = \frac{1}{4}\, n^2(n+1)^2$$

Arithmetic $\qquad S_n = \sum_{k=0}^{n-1}(a+kd) = \frac{n}{2}\,\{2a+(n-1)d\}$

Geometric (convergent for $-1 < r < 1$)

$$S_n = \sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}\ ,\ S_\infty = \frac{a}{1-r}$$

Binomial (convergent for $|x| < 1$)

$$(1+x)^n = 1 + nx + \frac{n!}{(n-2)!2!}x^2 + ... + \frac{n!}{(n-r)!r!}x^r + ...$$

where $\frac{n!}{(n-r)!r!} = \frac{n(n-1)(n-2)...(n-r+1)}{r!}$

Maclaurin series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + ... + \frac{x^k}{k!}f^{(k)}(0) + R_{k+1}$$

where $R_{k+1} = \frac{x^{k+1}}{(k+1)!}f^{(k+1)}(\theta x), \quad 0 < \theta < 1$

Taylor series

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + ... + \frac{h^k}{k!}f^{(k)}(a) + R_{k+1}$$

where $R_{k+1} = \frac{h^{k+1}}{(k+1)!}f^{(k+1)}(a+\theta h)\ ,\ 0 < \theta < 1.$

OR

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + ... + \frac{(x-x_0)^k}{k!}f^{(k)}(x_0) + R_{k+1}$$

where $R_{k+1} = \frac{(x-x_0)^{k+1}}{(k+1)!}f^{(k+1)}(x_0+(x-x_0)\theta), \quad 0 < \theta < 1$

Special Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^r}{r!} + ... \qquad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + ... \qquad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + \frac{(-1)^r x^{2r}}{(2r)!} + ... \qquad (\text{all } x)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + ... \qquad (|x| < \tfrac{\pi}{2})$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} +$$

$$... + \frac{1.3.5....(2n-1)}{2.4.6....(2n)}\frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ... + (-1)^n \frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\ell n(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... + (-1)^{n+1}\frac{x^n}{n} + ... \qquad (-1 < x \le 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ... + \frac{x^{2n+1}}{(2n+1)!} + ... \qquad (\text{all } x)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ... + \frac{x^{2n}}{(2n)!} + ... \qquad (\text{all } x)$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + ... \qquad (|x| < \tfrac{\pi}{2})$$

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} - \frac{1.3.5}{2.4.6}\frac{x^7}{7} +$$

$$... + (-1)^n \frac{1.3.5...(2n-1)}{2.4.6...2n}\frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + ... \frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

DERIVATIVES

function	derivative
x^n	nx^{n-1}
e^x	e^x
$a^x (a > 0)$	$a^x \ell n a$
$\ell n x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ell n a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\text{cosec } x$	$- \text{ cosec } x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$- \text{ cosec }^2 x$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\text{sech }^2 x$
$\text{cosech } x$	$- \text{ cosech } x \coth x$
$\text{sech } x$	$- \text{ sech } x \tanh x$
$\coth x$	$- \text{ cosech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x (x > 1)$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x (\|x\| < 1)$	$\dfrac{1}{1-x^2}$
$\coth^{-1} x (\|x\| > 1)$	$-\dfrac{1}{x^2-1}$

Product Rule

$$\frac{d}{dx}(u(x)v(x)) = u(x)\frac{dv}{dx} + v(x)\frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)\frac{du}{dx} - u(x)\frac{dv}{dx}}{[v(x)]^2}$$

Chain Rule

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \times g'(x)$$

Leibnitz's theorem

$$\frac{d^n}{dx^n}(f.g) = f^{(n)}.g + nf^{(n-1)}.g^{(1)} + \frac{n(n-1)}{2!}f^{(n-2)}.g^{(2)} + ... + \frac{n!}{(n-r)!r!}f^{(n-r)}.g^{(r)} + ... + f.g^{(n)}$$

INTEGRALS

function	integral
$f(x)\frac{dg(x)}{dx}$	$f(x)g(x) - \int \frac{df(x)}{dx} g(x) dx$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ell n\|x\|$ Note:- $\ell n\|x\| + K = \ell n\|x/x_0\|$
e^x	e^x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ell n\|\sec x\|$
cosec x	$-\ell n\|\text{ cosec } x + \cot x\|$ or $\ell n \left\|\tan \frac{x}{2}\right\|$
$\sec x$	$\ell n\|\sec x + \tan x\| = \ell n \left\|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right\|$
$\cot x$	$\ell n\|\sin x\|$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ell n\frac{a+x}{a-x}$ or $\frac{1}{a}\tanh^{-1}\frac{x}{a}$ $(\|x\| < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ell n\frac{x-a}{x+a}$ or $-\frac{1}{a}\coth^{-1}\frac{x}{a}$ $(\|x\| > a)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(a > \|x\|)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\frac{x}{a}$ or $\ell n\left(x + \sqrt{x^2+a^2}\right)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\frac{x}{a}$ or $\ell n\|x + \sqrt{x^2-a^2}\|$ $(\|x\| > a)$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ell n \cosh x$
cosech x	$-\ell n\,\|cosechx + cothx\|$ or $\ell n \left\|\tanh \frac{x}{2}\right\|$
sech x	$2\ \tan^{-1} e^x$
$\coth x$	$\ell n\|\sinh x\|$

Double integral

$$\int\int f(x,y)dxdy = \int\int g(r,s)Jdrds$$

where

$$J = \frac{\partial(x,y)}{\partial(r,s)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} \end{vmatrix}$$

LAPLACE TRANSFORMS

$$\tilde{f}(s) = \int_0^\infty e^{-st} f(t)dt$$

function	transform
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\dfrac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\dfrac{s}{s^2-\omega^2}$
$t \sin \omega t$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$
$t \cos \omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$H_a(t) = H(t-a)$	$\dfrac{e^{-as}}{s}$
$\delta(t)$	1
$e^{at} t^n$	$\dfrac{n!}{(s-a)^{n+1}}$
$e^{at} \sin \omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$
$e^{at} \sinh \omega t$	$\dfrac{\omega}{(s-a)^2-\omega^2}$
$e^{at} \cosh \omega t$	$\dfrac{s-a}{(s-a)^2-\omega^2}$

Let $\tilde{f}(s) = \mathcal{L}\{f(t)\}$ then

$$\begin{aligned}
\mathcal{L}\left\{e^{at} f(t)\right\} &= \tilde{f}(s-a), \\
\mathcal{L}\{t f(t)\} &= -\frac{d}{ds}(\tilde{f}(s)), \\
\mathcal{L}\left\{\frac{f(t)}{t}\right\} &= \int_{x=s}^{\infty} \tilde{f}(x) dx \text{ if this exists.}
\end{aligned}$$

Derivatives and integrals

Let $y = y(t)$ and let $\tilde{y} = \mathcal{L}\{y(t)\}$ then

$$\begin{aligned}
\mathcal{L}\left\{\frac{dy}{dt}\right\} &= s\tilde{y} - y_0, \\
\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} &= s^2\tilde{y} - sy_0 - y_0', \\
\mathcal{L}\left\{\int_{\tau=0}^{t} y(\tau) d\tau\right\} &= \frac{1}{s}\tilde{y}
\end{aligned}$$

where y_0 and y_0' are the values of y and dy/dt respectively at $t = 0$.

Time delay

Let $$g(t) = H_a(t) f(t-a) = \begin{cases} 0 & t < a \\ f(t-a) & t > a \end{cases}$$

then $$\mathcal{L}\{g(t)\} = e^{-as}\tilde{f}(s).$$

Scale change

$$\mathcal{L}\{f(kt)\} = \frac{1}{k}\tilde{f}\left(\frac{s}{k}\right).$$

Periodic functions

Let $f(t)$ be of period T then

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_{t=0}^{T} e^{-st} f(t) dt.$$

Convolution

Let $f(t) * g(t) = \int_{x=0}^{t} f(x)g(t-x)dx = \int_{x=0}^{t} f(t-x)g(x)dx$

then $\mathcal{L}\left\{f(t) * g(t)\right\} = \tilde{f}(s)\tilde{g}(s).$

RLC circuit

For a simple RLC circuit with initial charge q_0 and initial current i_0,

$$\tilde{E} = \left(r + Ls + \frac{1}{Cs}\right)\tilde{i} - Li_0 + \frac{1}{Cs}q_0.$$

Limiting values

initial value theorem

$$\lim_{t\to 0+} f(t) = \lim_{s\to\infty} s\tilde{f}(s),$$

final value theorem

$$\begin{aligned} \lim_{t\to\infty} f(t) &= \lim_{s\to 0+} s\tilde{f}(s), \\ \int_0^\infty f(t)dt &= \lim_{s\to 0+} \tilde{f}(s) \end{aligned}$$

provided these limits exist.

PROBABILITY DISTRIBUTIONS

Name	Parameters	Probability distribution / density function	Mean	Variance
Binomial	n, p	$P(X=r) = \frac{n!}{(n-r)!r!} p^r (1-p)^{n-r}$, $r = 0, 1, 2, ..., n$	np	$np(1-p)$
Poisson	λ	$P(X=n) = \frac{e^{-\lambda}\lambda^n}{n!}$, $n = 0, 1, 2, ...$	λ	λ
Normal	μ, σ	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\}$, $-\infty < x < \infty$	μ	σ^2
Exponential	λ	$f(x) = \lambda e^{-\lambda x}$, $x > 0, \quad \lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$